Name: _____

Instructor: _____

**Math 10560, Exam 3.**
**April 25, 2006**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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| 1. | (a) | (b) | (c) | (d) | (e) |
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| Multiple Choice | _____ |
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Multiple Choice

**1.**(7 pts.) Calculate

$$\lim_{n\to\infty} \frac{(\ln n)^2}{n}.$$

(a) 1 (b) $\infty$ (c) does not exist

(d) $e^2$ (e) 0

**2.**(7 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$.

(a) $\frac{5}{12}$ (b) $\frac{20}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{15}$ (e) $\frac{5}{4}$

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**3.**(7 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(a) diverges because $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$.

(b) diverges because the terms alternate.

(c) does not converge absolutely but does converge conditionally.

(d) diverges even though $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$.

(e) converges absolutely.

**4.**(7 pts.) Use Comparison Tests to determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$ (c) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$

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**5.**(7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\dfrac{x^2}{1+x}$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \; x^{n+2}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} x^{2n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$

**6.**(7 pts.) Find the degree 3 MacLaurin polynomial (Taylor polynomial centered at 0) for the function

$$\frac{e^x}{1-x^2}$$

(a) $1 + x + \dfrac{3x^2}{2} + \dfrac{7x^3}{6}$ (b) $1 + x - \dfrac{5x^3}{3}$ (c) $1 - \dfrac{x^2}{2} + \dfrac{x^3}{5}$

(d) $1 + x + \dfrac{x^2}{6} + 0\,x^3$ (e) $1 + x - \dfrac{x^3}{6}$

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**7.**(7 pts.) Which series below is a power series for $\cos(\sqrt{x})$ ?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{\,n}}{(2n)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

**8.**(7 pts.) Calculate

$$\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9}.$$

**Hint:** Without MacLaurin series this may be a long problem.

(a) $\frac{9}{7}$ (b) $0$ (c) $-\frac{1}{6}$ (d) $\infty$ (e) $\frac{7}{9}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

**9.**(11 pts.) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}}$$

converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

**Remark:** The correct answer with no justification is worth 2 points.

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**10.**(11 pts.) Use the Integral Test to discuss whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ converges.

**Remark:** Be sure to check that the Integral Test can be applied. The correct answer with no justification is worth 2 points.

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**11.**(11 pts.) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x-3)^n$$

**Remark:** The correct answer with no justification is worth 2 points.

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**12.**(11 pts.)

(a) Show that

$$\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that $|x| < 1$.

(b) Find

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

(**Hint:** First use term-by-term integration on the series in part (a).)

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